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Development of finite-amplitude disturbances in Poiseuille flow

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The process of three-dimensional distortion of previously two-dimensional disturbances was investigated in a rectangular channel. For the first time the three-dimensional structures at the breakdown station of the two-dimensional wave were studied by flow visualization. It was shown that the structures have forms identical with Λ-shaped vortices in a boundary layer of the flat plate. The spanwise spacing of the Λ-shaped vortices is quite independent of the mean-flow velocity and of the frequency of artificial disturbances.

1. Introduction

Systems with internal flow such as pipelines and channels are widespread in practice. Therefore investigation of the flow structure at transition to turbulence is a very important problem for such systems. The flow in a plane channel is usually considered for modelling and investigating the physical processes in these systems. This is related to the exact solution of the Navier–Stokes equations for the flow and to the simpler type of equations for disturbances (Goldshtic & Shtern 1977). Therefore many authors use this flow for studying the problem of three-dimensional growth of the disturbances (see Orszag & Patera 1981; Herbert 1981).

The process of laminar–turbulent transition in a channel was apparently investigated for the first time by Davies & White (1928). Further investigations had been carried out by Sherlin (1960), Narayanan & Narayana (1967) and Patel & Head (1969), who noted that the transition to turbulence began from a 'sudden' appearance of turbulent spots. Karnitz, Potter & Smith (1974) reduced the turbulence level to 0.3 %, which allowed them to observe sufficiently regular sinusoidal disturbances, followed by transition. The results on development of small artificial disturbances were obtained by Nishioka, Iida & Ichikawa (1975) and Kozlov & Ramazanov (1981), who confirmed the conclusions of linear theory of hydrodynamic stability for the plane Poiseuille flow. The investigations of Nishioka *et al.* (1975), Nishioka, Iida & Kanbayashi (1978) and Kozlov & Ramazanov (1980) for finite amplitude at subcritical and supercritical Reynolds numbers indicated the presence of spanwise periodic three-dimensional structures in the flow, which are similar to the structures defined by Klebanoff, Tidstrom & Sargent (1962) in a flat-plate boundary layer. In the recent experimental investigations by Kachanov & Levchenko (1982) and Kozlov, Levchenko & Saric (1983) carried out in a boundary layer on a flat plate, it was found that in the presence of subharmonic disturbances the laminar–turbulent transition can be realized by a resonance interaction of the fundamental 2-dimensional wave and a pair of oblique subharmonic ones.

The aim of the present study is to specify the form of the 3-dimensional structures

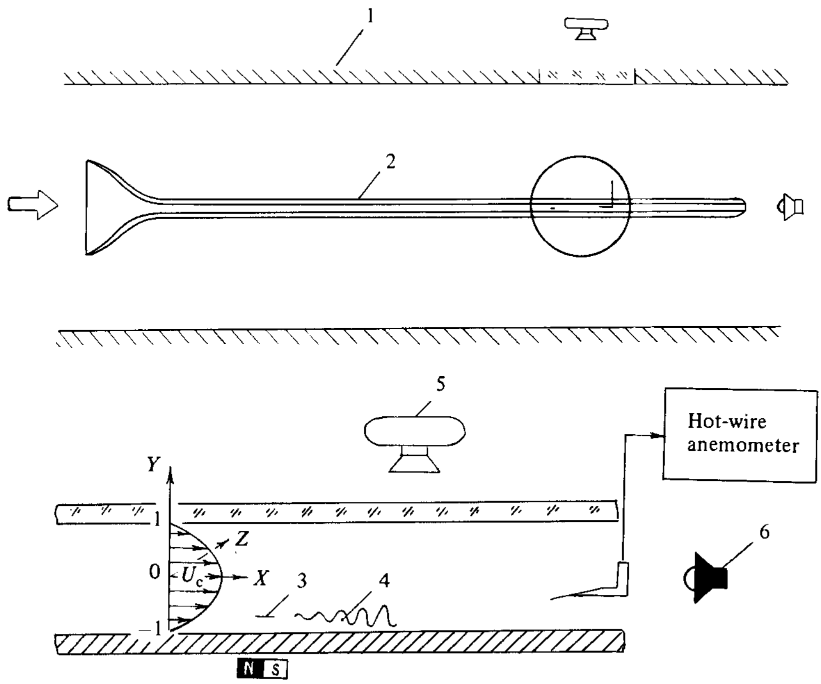

FIGURE 1. Schematic diagram of experiment: 1, test section; 2, channel; 3, ribbon; 4, smoke wire; 5, camera; 6, photoflash.

and to reveal the process of the 'breakdown' of the initially 2-dimensional wave for the case when the amplitude of subharmonic disturbances is negligibly small and the transition is of Klebanoff's type.

2. Experimental apparatus

The channel flow was studied in the installation described by Kozlov & Ramazanov (1981). Figure 1 shows a schematic diagram of the installation. The channel was made of Plexiglas, and its length, width and height were 4500, 15 and 400 mm respectively. This channel was placed in the test section of the low-turbulence wind tunnel T-324 of the Institute of Theoretical and Applied Mechanics of the USSR Academy of Sciences. The level of turbulence in the channel was reduced to approximately 0.1 % by means of a large-ratio (3:80) contraction at the inlet and low turbulence level of the free stream. As in Nishioka *et al.* (1975), disturbances were introduced into the flow by a metal ribbon of thickness 0.05 mm, width 3 mm and distance $X_0 = 3600$ mm from the inlet. This ribbon was placed in a constant magnetic field near one of the channel walls at a distance $Y = 0.85$ mm. Sinusoidal current passed through the ribbon with the frequency given by a wave generator. The frequency was checked by a frequency meter. The Reynolds number is based on the maximum velocity U_c in the channel and the channel half-width h. With the channel centre at $Y = 0$, wall dimensionless coordinates are 1 and -1 respectively.

As Kozlov & Ramazanov (1981) showed, plane Poiseuille flow can be produced in the whole measured region in absence of artificial finite amplitude disturbances.

The structure of the channel flow was visualized by the smoke-wire method (Dovgal *et al.* 1981). The essence of this method consists in placing a sufficiently thin wire into the flow. The wire is covered by mineral oil, for example. As soon as an electric current passes through the wire, accumulated drops of oil are burned down and generate thin

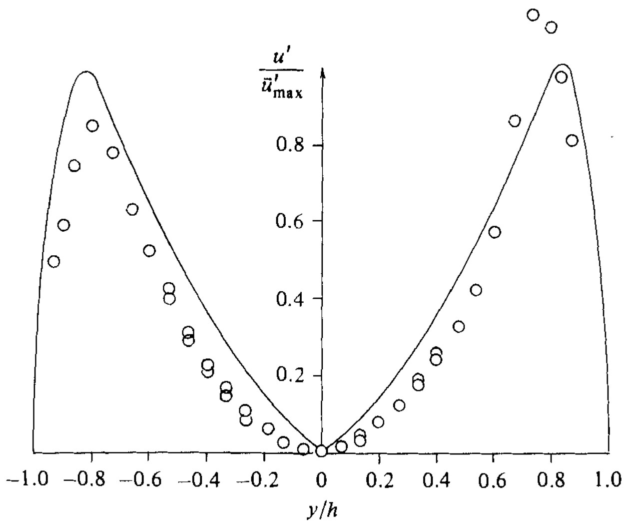

FIGURE 2. Amplitude distribution of u-fluctuations ($Re = 4000$): ○, data from Kozlov & Ramazanov (1981); ——, theory of Ito (from Nishioka *et al.* (1975).

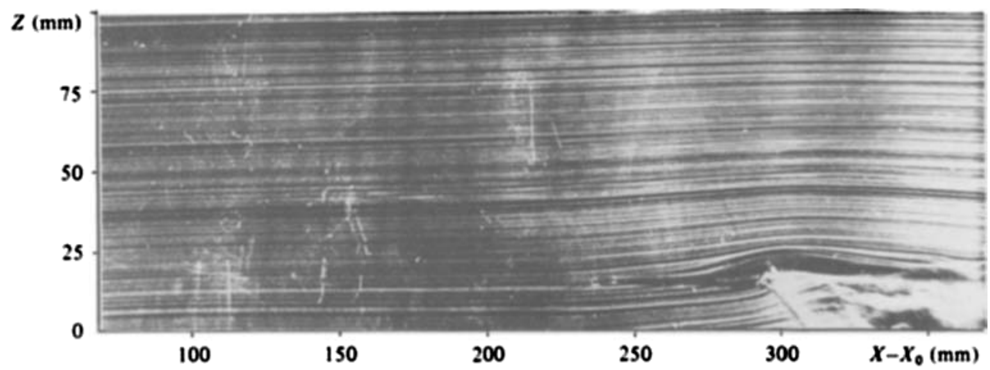

FIGURE 3. Visualization of flow field without artificial disturbances.

streaklines. The picture of streaklines give an imagination of the flow structure, even if it may be difficult to interpret in terms of the velocity field.

A schematic diagram of the visualization is shown in figure 1. A 68 μm diameter wire was located in the channel parallel to the long wall. Direct current of 20–25 V ran through the wire from a power supply. The smoke picture that appeared from burning the oil was lit by two photoflashes, located at the channel end and directed into the stream. The picture was taken through a window of the wind-tunnel test section and one of the channel walls. The second wall of the channel was black. The times for switching on the current and the channel lighting were separated by means of a delay line, so that smoke could propagate through the whole measuring region before the firing of the photoflashes. The wire oiling was done by a device that is described by Dovgal *et al.* (1981). The increase in the length of the wire at heating was compensated by a spring out of the channel.

The smoke wire was located close to the maximum of the Y-distribution of disturbance velocity, at a position 2.2 mm ($Y/h = 0.29$) from the wall, which corresponded approximately to the critical layer (see figure 2). As was shown by Hama

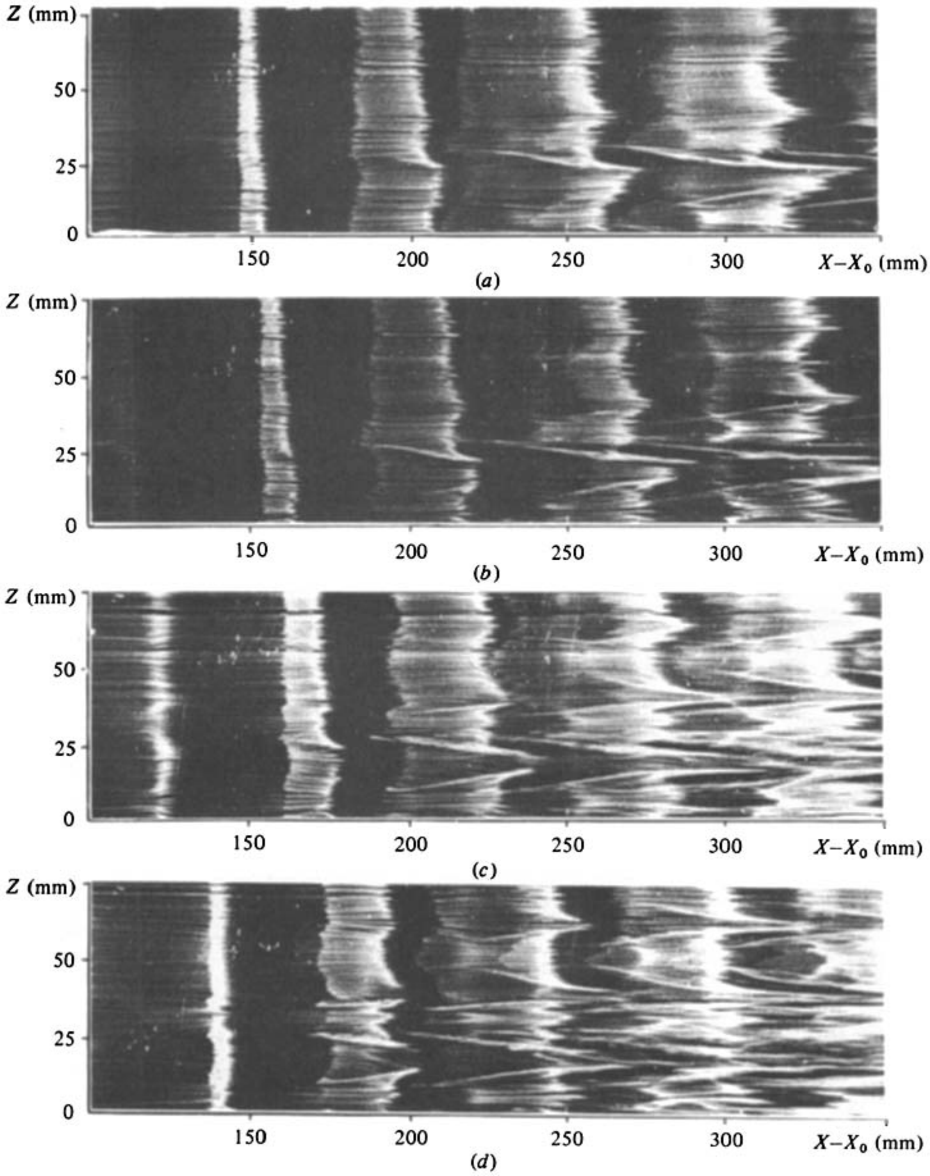

FIGURE 4. Development of three-dimensionality with increasing of disturbances amplitudes, $Re = 3850$, $f = 73$ Hz: (*a*) $u_{max}/U_c = 1.0\,\%$; (*b*) 1.1 %; (*c*) 1.26 %; (*d*) 1.36 %.

(1962), streaklines introduced into the flow at the critical layer distinguish more accurately a wave structure of the flow field, and the distance between the smoke accumulations coincides very well with the wavelength.

3. Results and discussion

Figure 3 shows the channel flow field in the absence of artificial disturbances at $Re = 3850$. Behind the wire there are barely visible periodic smoke accumulations with wavelength 8–9 mm, which were brought into the flow by switching on the smoke wire. These transient disturbances with a frequency 450–500 Hz do not distort the

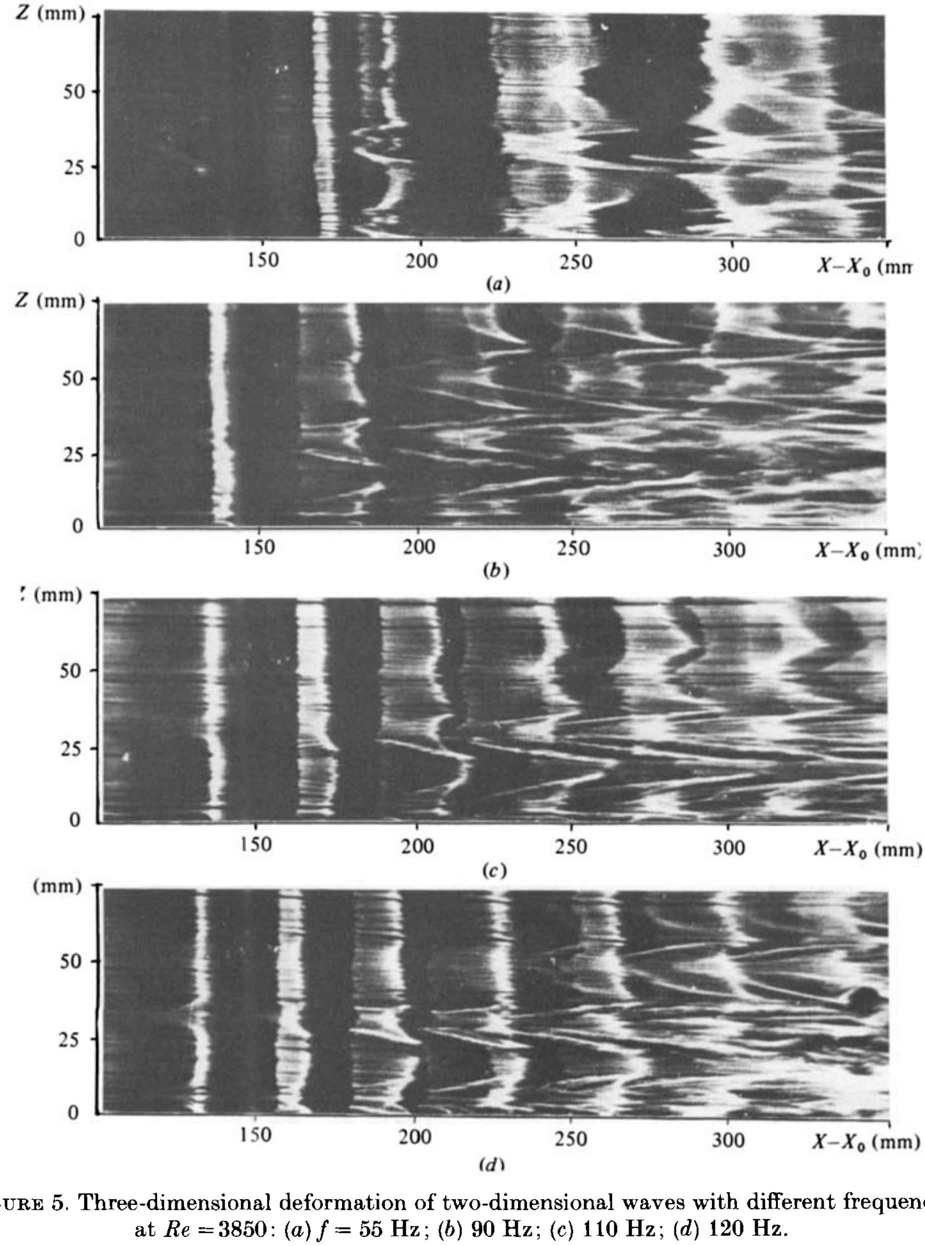

FIGURE 5. Three-dimensional deformation of two-dimensional waves with different frequencies at $Re = 3850$: (*a*) $f = 55$ Hz; (*b*) 90 Hz; (*c*) 110 Hz; (*d*) 120 Hz.

picture of the flow field that we wish to consider. A hot-wire probe that can be seen at the right lower corner of the photograph distorts the flow, but in the region of the sensitive hotwire ($X - X_0 = 235$ mm), distortions are practically absent. Therefore we consider the measurements with the hot-wire anemometer to be quite correct.

Figure 4 shows the three-dimensional distortion process of the initially two-dimensional wave front at frequency $f = 73$ Hz and $Re = 3850$. The amplitude, measured in the region of minimum along axis Z (see figure 7 and Nishioka *et al.* 1978) is 1 %–1.36 %. The flow direction is from left to right, $X - X_0$ is a distance from the vibrating ribbon, and $Z = 0$ corresponds to the middle of the channel. The bright stripes are observed at the left side of each photograph. These stripes are the smoke

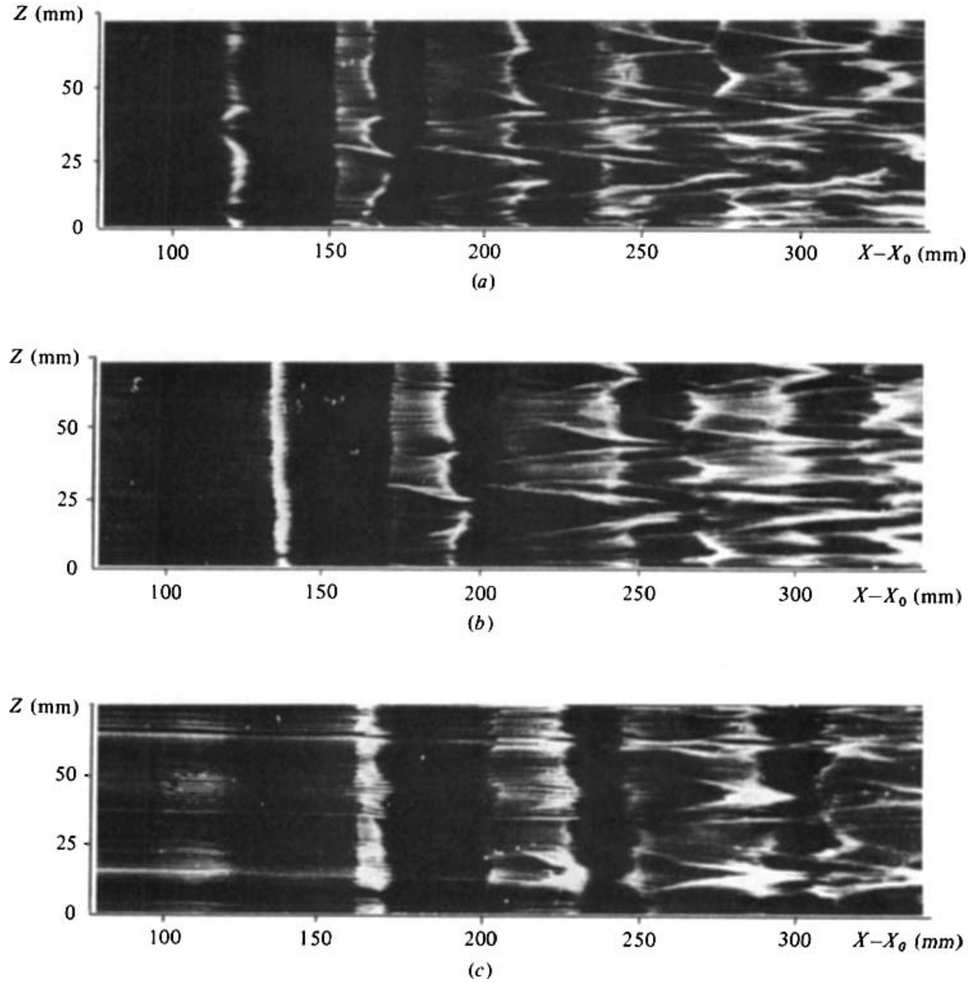

FIGURE 6. Appearance of Λ-shaped vortices at different Reynolds numbers: $f = 90$ Hz: (*a*) $Re = 3850$; (*b*) 4525; (*c*) 4830.

accumulations, which are sufficiently uniform in the Z-direction. As was shown by Hama (1962), streaklines that were propagated in the flow with the wave velocity were accumulated gradually in the regions of local decrease and began to turn back, forming a clearly visible wide strip. As the amplitude of the disturbance reached a value of 1 %, the double longitudinal vortices began to appear in the flow, the streaklines were accumulated in the region of the maximum vorticity forming two sloped lines in the form of the Greek letter Λ.

This Λ-shaped vortex may be clearly observed at the top photograph in the region $X - X_0 = 250$–300 mm. As long as the amplitude increases, the Λ-shaped vortex moves to the left (figures 4*b–d*). A chain of such vortex lines forms strictly one after another, the second and then the third chain of the Λ-shaped vortices appear beside the first. Thus the region of appearance of these Λ-shaped vortices moves upstream with increasing amplitude and also spreads from the centre of the channel to the periphery. Non-uniformity of the vortex appearance in the cross-section direction is associated with the fact that the vibrating ribbon was fixed at the edges. Therefore the vibration amplitude of the ribbon had a maximum at the centre and decreased

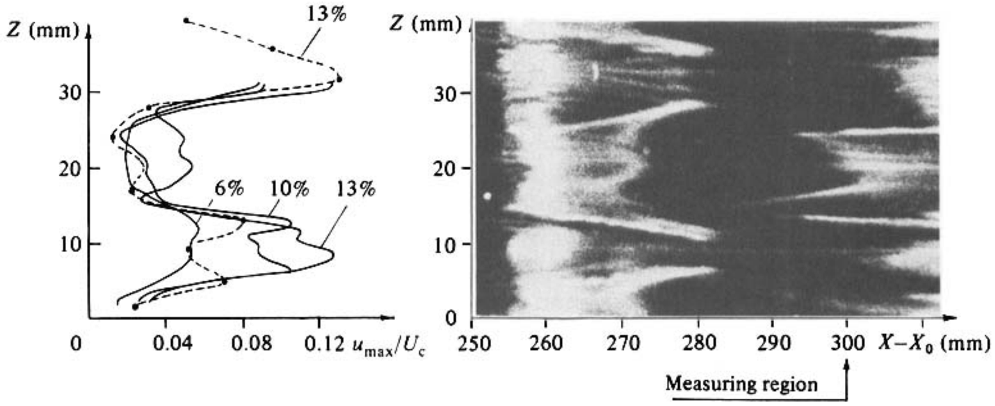

FIGURE 7. Variation of disturbance amplitude against Z-coordinate; $Y/h = 0.29$: ——, data from Nishioka *et al.* (1978); —●—, data of this study, $f = 72$ Hz.

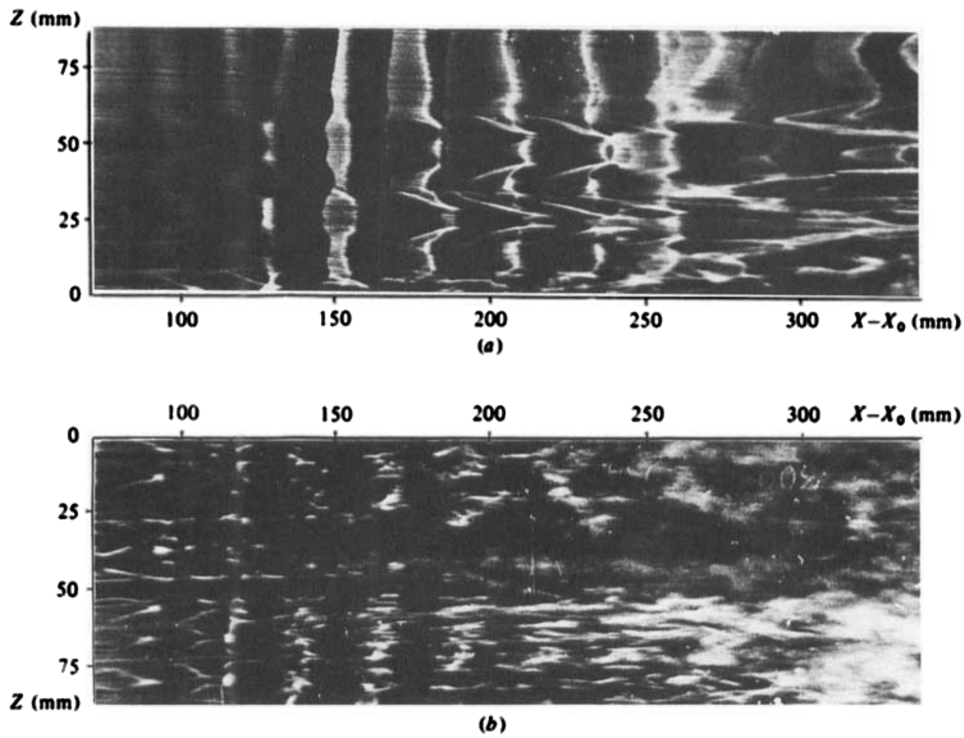

FIGURE 8. Vortex-structure duplication with increasing of disturbance amplitude; $Re = 3850$, $f = 150$ Hz: (*a*) small amplitude ($u_{max}/U_c \approx 1\,\%$); (*b*) large amplitude ($u_{max}/U_c \approx 10\,\%$).

towards the edges. Moreover, the disturbances were introduced near only one of the walls, so that the amplitude maxima are different at the two walls in figure 2. This asymmetry was also observed by Nishioka *et al* (1975), and led to the appearance of 'spikes' in the waveform of the fluctuation at only one of the channel walls; namely, that with the larger disturbance amplitude. At the same time, the wave front at the other wall had not been distorted.

Figure 5 shows the formation of Λ-shaped vortices for different frequencies. It is noteworthy that, in spite of considerable changes of the wavelength, the spanwise spacing of the Λ-shaped vortices remains constant and is approximately 23 mm. It does not change as the amplitude increases to a certain magnitude, and figure 4 confirms this. The mean-flow velocity does not change the arrangement of spacing of the Λ-shaped vortices. Figures 6 compares the flow field for various Reynolds numbers, and confirms our conclusion.

Apparently, the wavelength along the Z-axis is a certain invariant which depends on the channel's width but is independent of the flow parameters. A channel with similar size was used by Nishioka *et al.* (1978), and the disturbance irregularity in the Z-direction had the same wavelength (see figure 7).

Figure 8 shows the transition to three-dimensionality of a previously two-dimensional wave with frequency 150 Hz at $Re = 3850$. The top photograph shows the wave development at sufficiently low amplitude, approximately 1 %, and shows no principal differences from the previous photographs. The bottom photograph displays the process of the transition to three-dimensionality at sufficiently high amplitude (more than 10 %) measured at a local minimum in the Z-direction. It is obvious that the vortex forms differ strongly from the Λ-shaped vortex, and the wavelength along the Z-axis is half that of a low amplitude. Perhaps this phenomenon is connected with the formation of the Λ-shaped vortices at the opposite wall at the same place and with their influence on the transition to three-dimensionality.

5. Conclusions

Finally, we conclude the following.

(1) The transition to turbulence happens in a spatial development of the hydrodynamic stability waves like Tollmien–Schlichting waves, via three-dimensional distortions, the formation of Λ-shaped vortices, and their breakdown into turbulence, i.e. in the same way as in the experiments of Klebanoff *et al.* (1962).

(2) The spanwise size of a Λ-shaped vortex seems to be an invariant, independent of frequency of the disturbances, and the parameters of the flow, but apparently dependent on the channel width.

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